## Notation

| " . " | event (in probability) |
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| $\{\cdots\}$ | set |
| \| $\cdot 1$ | absolute value of a number, or cardinality (number of elements) of a set, or determinant of a matrix |
| $\\|\cdot\\|^{2}$ | square of the norm; sum of the squared components of a vector |
| [.] | floor; largest integer which is not larger than the argument |
| $[a, b]$ | the interval of real numbers from $a$ to $b$ |
| 【.】 | evaluates to 1 if argument is true, and to 0 if it is false |
| $\nabla$ | gradient operator, e.g., $\nabla E_{\text {in }}$ (gradient of $E_{\text {in }}(\mathbf{w})$ with respect to w) |
| (.) ${ }^{-1}$ | inverse |
| $(\cdot)^{\dagger}$ | pseudo-inverse |
| $(\cdot)^{\mathrm{T}}$ | transpose (columns become rows and vice versa) |
| $\binom{N}{k}$ | number of ways to choose $k$ objects from $N$ distinct objects (equals $\frac{N!}{(N-k)!k!}$ where '!' is the factorial) |
| $A \backslash B$ | the set $A$ with the elements from set $B$ removed |
| 0 | zero vector; a column vector whose components are all zeros |
| $\{1\} \times \mathbb{R}^{d}$ | $d$-dimensional Euclidean space with an added 'zeroth coordinate' fixed to 1 |
| $\epsilon$ | tolerance in approximating a target |
| $\delta$ | bound on the probability of exceeding $\epsilon$ (the approximation tolerance) |
| $\eta$ | learning rate (step size in iterative learning, e.g., in stochastic gradient descent) |
| $\lambda$ | regularization parameter |
| $\lambda_{C}$ | regularization parameter corresponding to weight budget C |
| $\Omega$ | penalty for model complexity; either a bound on generalization error, or a regularization term |
| $\theta$ | logistic function $\theta(s)=e^{s} /\left(1+e^{s}\right)$ |
| $\Phi$ | feature transform, $\mathbf{z}=\Phi(\mathbf{x})$ |
| $\Phi_{\mathrm{Q}}$ | $Q$ th-order polynomial transform |


| $\phi$ | a coordinate in the feature transform $\Phi, z_{i}=\phi_{i}(\mathbf{x})$ |
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| $\mu$ | probability of a binary outcome |
| $\nu$ | fraction of a binary outcome in a sample |
| $\sigma^{2}$ | variance of noise |
| $\mathcal{A}$ | learning algorithm |
| $\operatorname{argmin}_{a}(\cdot)$ | the value of $a$ at which the minimum of the argument is achieved |
| $\mathcal{B}$ | an event (in probability), usually 'bad' event |
| $b$ | the bias term in a linear combination of inputs, also called |
|  | $w_{0}$ |
| bias | the bias term in bias-variance decomposition |
| $B(N, k)$ | maximum number of dichotomies on $N$ points with a break point $k$ |
| C | bound on the size of weights in the soft order constraint |
| ${ }_{\sim}^{\text {d }}$ | dimensionality of the input space $\mathcal{X}=\mathbb{R}^{d}$ or $\mathcal{X}=\{1\} \times \mathbb{R}^{d}$ |
| $\tilde{d}$ | dimensionality of the transformed space $\mathcal{Z}$ |
| $d_{\mathrm{vC}}, d_{\mathrm{vc}}(\mathcal{H})$ | VC dimension of hypothesis set $\mathcal{H}$ |
| D | data set $\mathcal{D}=\left(\mathbf{x}_{1}, y_{1}\right), \cdots,\left(\mathbf{x}_{N}, y_{N}\right)$; technically not a set, but a vector of elements $\left(\mathbf{x}_{n}, y_{n}\right)$. $\mathcal{D}$ is often the training set, but sometimes split into training and validation/test sets. |
| $\mathcal{D}_{\text {train }}$ | subset of $\mathcal{D}$ used for training when a validation or test set is used. |
| $\mathcal{D}_{\text {val }}$ | validation set; subset of $\mathcal{D}$ used for validation. |
| $E(h, f)$ | error measure between hypothesis $h$ and target function $f$ |
| $e^{x}$ | exponent of $x$ in the natural base $e=2.71828$. |
| $\mathrm{e}(h(\mathbf{x}), f(\mathbf{x}))$ | pointwise version of $E(h, f)$, e.g., $(h(\mathbf{x})-f(\mathbf{x}))^{2}$ |
| $\mathrm{e}_{n}$ | leave-one-out error on example $n$ when this $n$th example is excluded in training [cross validation] |
| $\mathbb{E}[\cdot]$ | expected value of argument |
| $\mathbb{E}_{\mathbf{x}}[\cdot]$ | expected value with respect to $\mathbf{x}$ |
| $\mathbb{E}[y \mid \mathbf{x}]$ | expected value of $y$ given $\mathbf{x}$ |
| $E_{\text {aug }}$ | augmented error (in-sample error plus regularization term) |
| $E_{\text {in }}, E_{\text {in }}(h)$ | in-sample error (training error) for hypothesis $h$ |
| $E_{\text {cv }}$ | cross validation error |
| $E_{\text {out }}, E_{\text {out }}(h)$ | out-of-sample error for hypothesis $h$ |
| $E_{\text {out }}^{\text {D }}$ | out-of-sample error when $\mathcal{D}$ is used for training |
| $\bar{E}_{\text {out }}$ | expected out-of-sample error |
| $E_{\text {val }}$ | validation error |
| $E_{\text {test }}$ | test error |
| $f$ | target function, $f: \mathcal{X} \rightarrow \mathcal{Y}$ |
| $g$ | final hypothesis $g \in \mathcal{H}$ selected by the learning algorithm; $g: \mathcal{X} \rightarrow \mathcal{Y}$ |
| $g^{(\mathcal{D})}$ | final hypothesis when the training set is $\mathcal{D}$ |
| $\bar{g}$ | average final hypothesis [bias-variance analysis] |


| $g^{-}$ | final hypothesis when trained using $\mathcal{D}$ minus some points |
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| g | gradient, e.g., $\mathbf{g}=\nabla E_{\text {in }}$ |
| $h$ | a hypothesis $h \in \mathcal{H} ; h: \mathcal{X} \rightarrow \mathcal{Y}$ |
| $\tilde{h}$ | a hypothesis in transformed space $\mathcal{Z}$ |
| $\mathcal{H}$ | hypothesis set |
| $\mathcal{H}_{\text {I }}$ | hypothesis set that corresponds to perceptrons in $\Phi$ transformed space |
| $\mathcal{H}(C)$ | restricted hypothesis set by weight budget $C$ [soft order constraint] |
| $\mathcal{H}\left(\mathrm{x}_{1}, \ldots, \mathrm{x}_{N}\right)$ | dichotomies (patterns of $\pm 1$ ) generated by $\mathcal{H}$ on the points $\mathbf{x}_{1}, \cdots, \mathbf{x}_{N}$ |
| H | The hat matrix [linear regression] |
| I | identity matrix; square matrix whose diagonal elements are 1 and off-diagonal elements are 0 |
| K | size of validation set |
| $L_{q}$ | $q$ th-order Legendre polynomial |
| $\ln$ | logarithm in base $e$ |
| $\log _{2}$ | logarithm in base 2 |
| M | number of hypotheses |
| $m_{\mathcal{H}}(N)$ | the growth function; maximum number of dichotomies generated by $\mathcal{H}$ on any $N$ points |
| $\max (\cdot, \cdot)$ | maximum of the two arguments |
| $N$ | number of examples (size of $\mathcal{D}$ ) |
| $o(\cdot)$ | absolute value of this term is asymptotically negligible compared to the argument |
| $O(\cdot)$ | absolute value of this term is asymptotically smaller than a constant multiple of the argument |
| $P(\mathbf{x})$ | (marginal) probability or probability density of $\mathbf{x}$ |
| $P(y \mid \mathbf{x})$ | conditional probability or probability density of $y$ given $\mathbf{x}$ |
| $P(\mathbf{x}, y)$ | joint probability or probability density of $\mathbf{x}$ and $y$ |
| $\mathbb{P}[\cdot]$ | probability of an event |
| $Q$ | order of polynomial transform |
| $Q_{f}$ | complexity of $f$ (order of polynomial defining $f$ ) |
| R | the set of real numbers |
| $\mathbb{R}^{d}$ | $d$-dimensional Euclidean space |
| $s$ | signal $s=\mathbf{w}^{\mathrm{T}} \mathbf{x}=\sum_{i} w_{i} x_{i}$ ( $i$ goes from 0 to $d$ or 1 to $d$ depending on whether $\mathbf{x}$ has the $x_{0}=1$ coordinate or not) |
| $\operatorname{sign}(\cdot)$ | sign function, returning +1 for positive and -1 for negative |
| $\sup _{a}($. | supremum; smallest value that is $\geq$ the argument for all $a$ |
| T | number of iterations, number of epochs |
| $t$ | iteration number or epoch number |
| $\tanh (\cdot)$ | hyperbolic tangent function; $\tanh (s)=\left(e^{s}-e^{-s}\right) /\left(e^{s}+e^{-s}\right)$ |
| trace(•) | trace of square matrix (sum of diagonal elements) |
| V | number of subsets in $V$-fold cross validation ( $V \times K=N$ ) |
| v | direction in gradient descent (not necessarily a unit vector) |


| $\hat{\mathbf{v}}$ | unit vector version of $\mathbf{v}$ [gradient descent] |
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| var | the variance term in bias-variance decomposition |
| w | weight vector (column vector) |
| w | weight vector in transformed space $\mathcal{Z}$ |
| $\hat{\mathbf{w}}$ | selected weight vector [pocket algorithm] |
| $\mathrm{w}^{*}$ | weight vector that separates the data |
| $\mathrm{w}_{\text {lin }}$ | solution weight vector to linear regression |
| $\mathbf{w}_{\text {reg }}$ | regularized solution to linear regression with weight decay |
| $\mathbf{w}_{\text {PLA }}$ | solution weight vector of perceptron learning algorithm |
| $w_{0}$ | added coordinate in weight vector $\mathbf{w}$ to represent bias $b$ |
| x | the input $\mathrm{x} \in \mathcal{X}$. Often a column vector $\mathrm{x} \in \mathbb{R}^{d}$ or $\mathbf{x} \in$ $\{1\} \times \mathbb{R}^{d} . x$ is used if input is scalar. |
| $x_{0}$ | added coordinate to $\mathbf{x}$, fixed at $x_{0}=1$ to absorb the bias term in linear expressions |
| $\mathcal{X}$ | input space whose elements are $\mathbf{x} \in \mathcal{X}$ |
| X | matrix whose rows are the data inputs $\mathbf{x}_{n}$ [linear regression] |
| XOR | exclusive OR function (returns 1 if the number of 1 's in its input is odd) |
| $y$ | the output $y \in \mathcal{Y}$ |
| y | column vector whose components are the data set outputs $y_{n}$ [linear regression] |
| $\hat{\mathbf{y}}$ | estimate of y [linear regression] |
| $\mathcal{Y}$ | output space whose elements are $y \in \mathcal{Y}$ |
| $\mathcal{Z}$ | transformed input space whose elements are $\mathbf{z}=\Phi(\mathbf{x})$ |
| Z | matrix whose rows are the transformed inputs $\mathbf{z}_{n}=\Phi\left(\mathbf{x}_{n}\right)$ [linear regression] |

